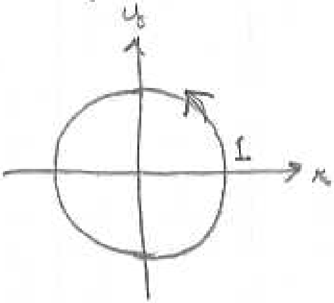
Page 855 #9

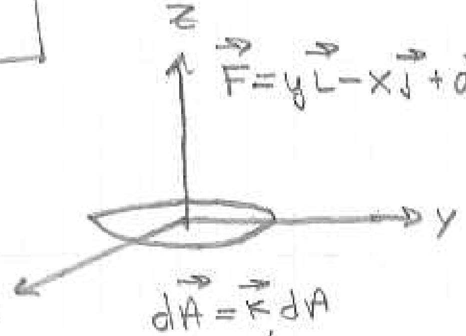
$\vec{F} = y\vec{i} - x\vec{j}$



Circulation (Greens theorem)

$$\int_C \vec{F}\cdot d\vec{r} = \int_C \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dx\,dy$$

$$= \int -2\,dx\,dy = -2A = -2\pi\cdot 1^2 = \boxed{-2\pi}$$



$\vec{F} = y\vec{i} - x\vec{j} + 0\vec{k}$

Circulation (Stokes theorem)

$d\vec{A} = \vec{k}\,dA$

$$\int_C \vec{F}\cdot d\vec{r} = \int_R \text{curl}\,\vec{F}\cdot d\vec{A}$$

curl F:

$$\nabla\times\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y & -x & 0 \end{vmatrix}$$

$$= \vec{i}(0) - \vec{j}(0) + \vec{k}(-1-1)$$

$$= -2\vec{k}$$

$$= \int_R (-2\vec{k})(\vec{k}\,dA)$$

$$= \int_R -2\,dA$$

$$= -2A = \boxed{-2\pi}$$