Excellent job!

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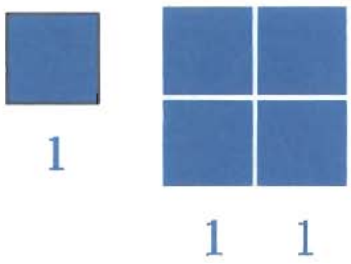
Matt Bacon

A **fractal** is a rough or fragmented geometric shape, often occurring in nature, that can be subdivided in parts, each of which is a reduced-size copy of the whole.

The fractal fern provides an excellent example, where each leaf is composed of the original at a reduced size. A fractal dimension is an exponent that can be used to predict the characteristics of a fractal. When you multiply the base of a fractal by its fractal detention, you will get its area. Mathematically it is described as:

$$d = \frac{\text{Log (number of self similar pieces)}}{\text{Log (magnification factor in base)}}$$

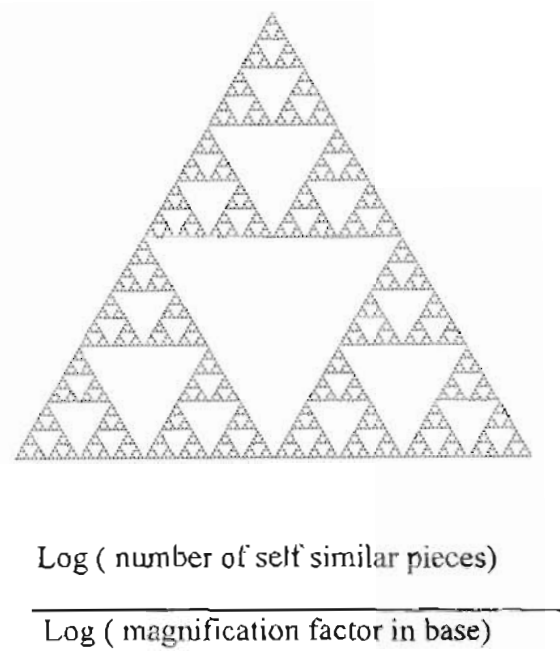
Ex. If we were to take a square where S = 1, and then create a similar square where S = 2, we would create 4 of the original squares in doing so.



$$d = \frac{\text{Log (4)}}{\text{Log (2)}} = 2$$

And so, $S^d = 2^2 = 4$

If we examine the three larger triangles in this Sierpinski triangle, we can see that from the original triangle we generate three new triangles, half the size of the original base. If we use these numbers we can find the fractal dimension of the Sierpinski triangle.



$$d = \frac{\text{Log (number of self similar pieces)}}{\text{Log (magnification factor in base)}}$$

$$d = \frac{\log (3)}{\log (2)}$$

$$d = 1.5850$$

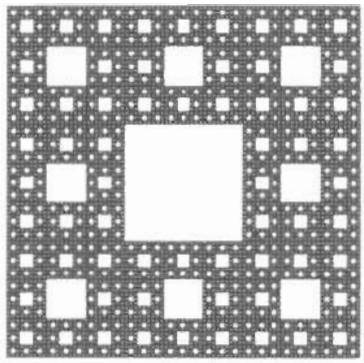
If we try this same process using triangle generated from the next stage, we have nine triangles, each a quarter the size of the original base.

$$d = \frac{\text{Log (number of self similar pieces)}}{\text{Log (magnification factor in base)}}$$

$$d = \frac{\log (9)}{\log (4)}$$

$$d = 1.5850$$

This process will also work for determining the fractal dimension of the Sierpinski carpet and the Sierpinski pyramid.

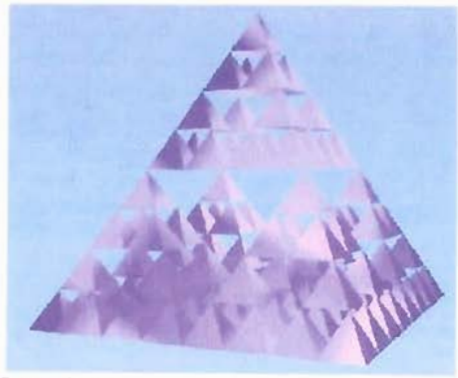


$$d = \frac{\text{Log (number of self similar pieces)}}{\text{Log (magnification factor in base)}}$$

$$d = \frac{\log (8)}{\log (3)}$$

$$d = 1.8928$$

In the Sierpinski pyramid, five new pyramids are generated from the original and the base is composed of four of the pyramids.



$$d = \frac{\text{Log (number of self similar pieces)}}{\text{Log (magnification factor in base)}}$$

$$d = \frac{\log (5)}{\log (4)}$$

$$d = 1.1610$$

1) A piece of broccoli generates 13 new stalks, one-seventh the size of the original. Find the fractal dimension.

$$d = \frac{\text{Log (number of self similar pieces)}}{\text{Log (magnification factor in base)}}$$

$$d = \frac{\log (13)}{\log (7)}$$

$$d = 1.3181$$

2) While battling the Lernean Hydra, Hercules realizes that every time he cuts of one of it's heads, two grow back at one third the size of the original. Find the fractal dimension of the Hydra's heads.

$$d = \frac{\text{Log (number of self similar pieces)}}{\text{Log (magnification factor in base)}}$$

$$d = \frac{\log (2)}{\log (3)}$$

$$d = 0.6309$$